RAN-2103000206020033

T.Y.B.Sc. (Sem. VI) Examination April - 2023

Mathematics : Paper-MTH-603 - Real Analysis - III (New Course)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(૧) નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **T.Y.B.Sc. (Sem. VI)**

Name of the Subject :

☛ **Mathematics : Paper-MTH-603 - Real Analysis - III (New Course)**

Subject Code No.: **2103000206020033**

Seat No.: | | | | | |

Student's Signature

(2) Figures to the right indicate marks of the question.

(3) Follow usual notations and conventions.

Q-1 Answer the following questions : (Any five). **[10]**

(1) Prove that the sequence { 1,1,1,1,... } is (c, 1) summable.

(2) Define : (c, 1) summability of sequence.

(3) If $f_n(x) = \frac{x}{n} e^{-x/n}$ $(0 \leq x < \infty)$. does the sequence $\{f_n\}_{n=1}^{\infty}$ converges uniformly to 0 on [0,500].

(4) Define : pointwise convergence of a sequence.

(5) Define : sets of measure zero and prove that the set{1,2,3} has measure zero.

(6) Define : upper integral of a function.

(7) Find $\lim\limits_{n\to\infty} \frac{1}{n} [e^{3/n} + e^{6/n} + e^{9/n} + ... + e^{3n/n}]$

(8) If $f(x) = \int_0^x \sqrt{t + t^6}\, dt$ $(x > 0)$ then find $F'(1)$

Q-2 Attempt Any Two: **[10]**

(a) If s_1, s_2, s_3, ... is $(c, 1)$ summable to S, and if $t \in R$ then prove that t, s_1, s_2, s_3, ... is $(c, 1)$ summable to S.

(b) Let $\{s_n\}_{n=1}^{\infty}$ be a monotone sequence then prove that $\{\sigma_n\}_{n=1}^{\infty}$ is a monotone sequence where $\sigma_n = \frac{s_1 + s_2 + ... + s_n}{n}$.

(c) Prove that a sequence that diverges to infinity cannot be $(c, 1)$ summable.

Q-3 Attempt Any Two: [10]

(a) Let $\{f_n\}_{n=1}^{\infty}$ be sequence of real valued functions on a set E. Then prove that $\{f_n\}_{n=1}^{\infty}$ is uniformly convergent on E if and only if given $\varepsilon > 0 \; \exists \; N \in I$ such that $|f_n(x) - f(x)| < \varepsilon$, m, n $\geq N$, $x \in$ E.

(b) Define point wise convergence of sequence of functions.

If $g_n(x) = \frac{x}{1+nx}$ $(0 \leq x < \infty)$ then show that .

the sequence $\{g_n(x)\}_{n=1}^{\infty}$ converges point wise .

(c) The sequence of function $\{f_n\}_{n=1}^{\infty}$ converges to f on E then prove that $l.u.\, b_{x \in E} |f_n(x) - f(x)| \to 0$.

Q-4 Attempt Any Two : [10]

(a) Prove that any lower sum cannot exceed any upper sum .

(b) If each of the subsets $E_1, E_2, E_3, ...$ of R^1 is of measure zero then prove that $\bigcup_{n=1}^{\infty} E_n$ is also of measure zero.

(c) If $f \in R\,[a,b]$ and λ is any real number then prove that λ f $\in R\,[a,b]$ and $\int_a^b (\lambda f) = \lambda \int_a^b f$. Where $(\lambda > 0)$.

Q-5 Attempt Any Two: [10]

(a) If f is continuous on a closed bounded interval $[\, a, b\,]$ and if $F(x) = \int_a^x f(t)dt \; (a \leq x \leq b)$ then prove that $F'(x) = f(x), (a \leq x \leq b)$.

(b) State and prove first mean value theorem for integrals .

(c) Let ϕ be a real valued function on the closed bounded interval $[\, a, b\,]$ such that ϕ is continuous on $[\, a, b\,]$. Let $\phi(a) = A$, $\phi(b) = B$. Then f is continuous on $\phi\,[\, a, b\,]$ and prove that

$$\int_A^B f(x)\, dx = \int_a^b f[\phi(u)]\, \phi'(u)\, du.$$
